

STUDY ON DIFFERENT TYPES OF TOPOLOGIES
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ABSTRACT

The previous section is evolved with the concept of topology on the edge set. Thus we can study the different types of topologies with respect to the set of edges. This section, will throw light on the following types of topologies: stronger and weaker, not comparable, confinite or finite complement topology, co-countable topology, intersection and union of topology, hereditary topology etc. Further this concept will be utilized in some theorems also.

Key words: Topology, hereditary

INTRODUCTION

At the present time the word 'topology' is being commonly used and getting popularity day by day in the filed of modern mathematics while in the last century when India failed in her trial of being free from the British rule, the word topology was found on the tongue of rare mathematician. *The word topology seems to be derived from Greek wors: 'topo' means 'a place' and 'logo' means 'a discourse'.* The meaning of topology is 'by associating the things with particular place or Town. The use of word 'topology' first occurred in the title of the book 'vorstudien Zur Topologic by listing in 1847. The general topology got its real start in 1906 due to Riesz, Frechet and Moore. Actually, it is supposed to be the branch of geometry, dealing with the properties which are unaffected by changes in shapes and size.

TYPES OF TOPOLOGIES

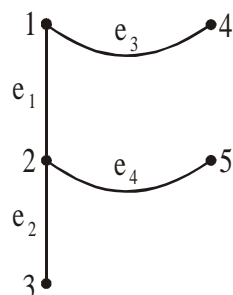
The previous section is evolved with the concept of topology on the edge set. Thus we can study the different types of topologies with respect to the set of edges. This section, will throw light on the following types of topologies: stronger and weaker, not comparable, confinite or finite complement topology, co-countable topology, intersection and union of topology, hereditary topology etc. Further these concepts will be utilized in some theorems also.

STRONGER AND WEAKER TOPOLOGIES:

Two topologies T_1 and T_2 defined on the edge set E is said to be weaker and stronger topology if $T_1 \sqsubset T_2$, then T_1 is said to weaker topology than T_2 and T_2 is said to be finer or stronger topology than T_1 let us take an example to verify the above given definition.

Example: If the edge set $E = \{e_1, e_2, e_3, e_4\}$, then define two topologies on E .

$$T_1 = \{\phi, \{e_1\}, E\} \text{ and } T_2 = \{\phi, \{e_1\}, \{e_3, e_4\}, \{e_1, e_3, e_4\}, E\}$$

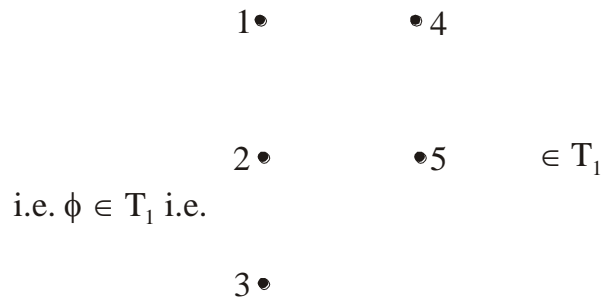


In the example the edge set E can be shown graphically:

As it is already defined that T_1 and T_2 are the topologies, thus it has to shown that either $T_1 \subset T_2$, or $T_2 \subset T_1$, to say that these are weaker or stronger topologies.

$$T_1 = \{\phi, E, \{e_1\}\}, T_2 = \{\phi, \{e_1\}, \{e_2, e_3\}, \{e_1, e_2, e_3\}, E\}$$

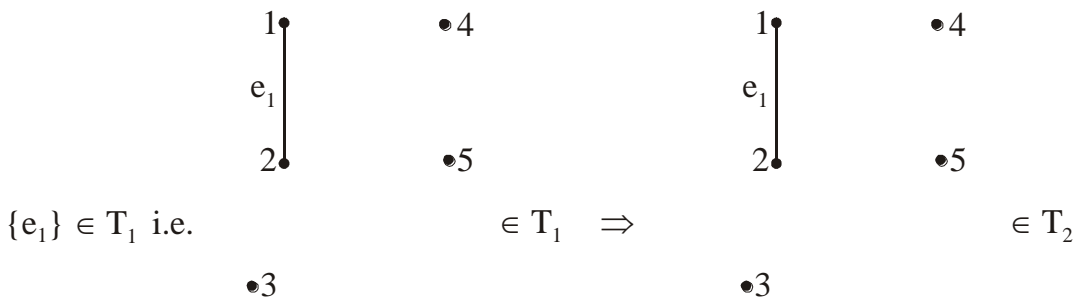
If T_1 is the subset of T_2 , then every edge subset of T_1 must belong to T_2 .



Thus φ ∈ T₂ hence φ ∈ T₁ ⇒ φ ∈ T₂

Similarly,
 ⇒ 2• •5 ∈ T₂

3•



also {e₁} ∈ T₁ ⇒ {e₁} ∈ T₂

Similarly, we can observe that the subset E itself lies in T₂.

Thus we can say that T₁ is contained in T₂. Hence T₁ is a weaker topology and T₂ is stronger topology defined on the edge set E.

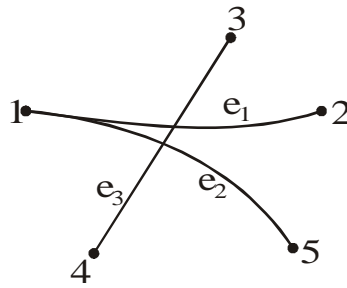
NOT COMPARABLE TOPOLOGIES:

Two topologies T₁ and T₂ defined on the edge set E on the graph G is said to be noncomparable topologies if T₁ ⊈ T₂ and T₂ ⊈ T₁ i.e. T₁ is not contained in T₂ and T₂ is not contained in T₁.

let us consider an edge set E = {e₁, e₂, e₃} where T₁ and T₂ are the two topologies such that

$$T_1 = \{\phi, \{e_1, e_2\}, E\} \text{ and } T_2 = \{\phi, \{e_1, e_3\}, E\}.$$

Now to show that T₁ and T₂ are non-comparable, we have to show that

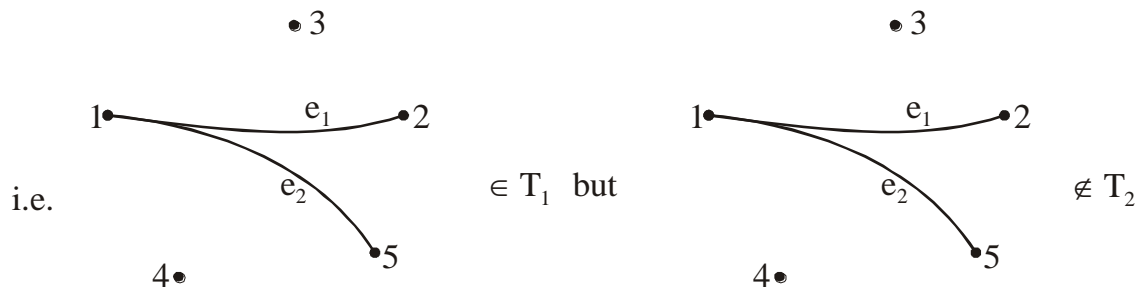


$T_1 \not\subset T_2$ and $T_2 \not\subset T_1$.

If T_1 and T_2 are not contained in each other, then all the edge subsets of T_1 and T_2 are not the number of either.

First to show $T_1 \not\subset T_2$

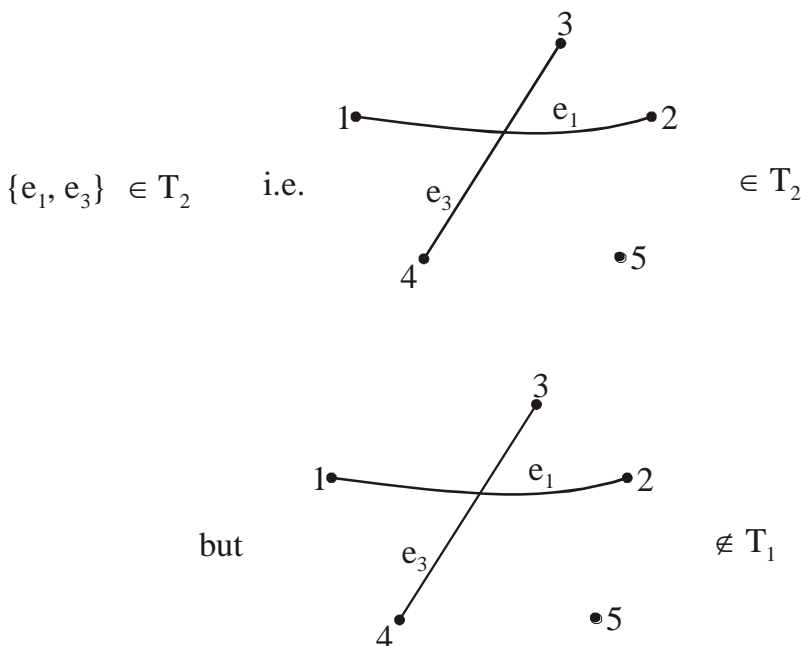
$\{e_1, e_2\} \in T_1$



Thus all the edge subsets of T_1 does not belong to T_2 .

Hence $T_1 \not\subset T_2$

Now to verify $T_2 \not\subset T_1$



Hence $T_2 \not\subset T_1$

Hence both the topologies are not contained in each other. Thus topologies T_1 and T_2 are non-comparable on the edge set E .

TRIVIAL TOPOLOGIES:

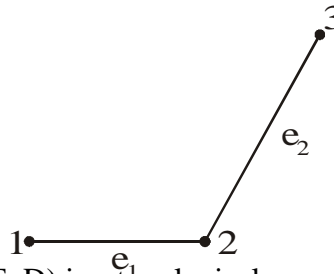
There are two kind of trivial topologies:

DISCRETE TOPOLOGY: For a given non-empty set E and D a family of all the subsets defined over the edge set E , so (E, D) is a topological space, as D is a discrete topology on E .

Let us understand the concept on any edge set.

EXAMPLE: let us take an edge set $E = \{e_1, e_2\}$ s.t. where D is a class of all the edge subsets of the set E .

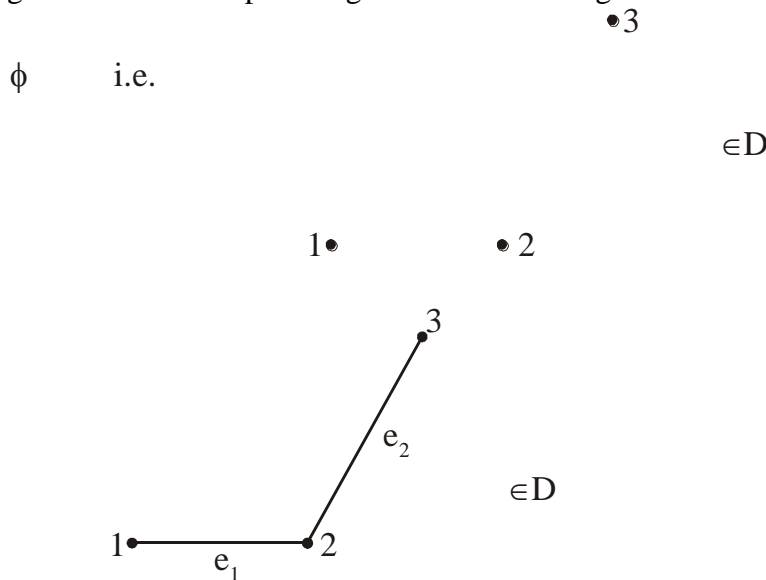
i.e. $D = \{\emptyset, \{e_1\}, \{e_2\}, E\}$



Verify that the class D is discrete topology and (E, D) is a topological space.

Sol. : Any topology is said to be discrete topology if it contains all the edge subset of any edge set. So as the class D is containing all the edge subset of the edge set E , it is needed to satisfy that the class D is a topology. We will satisfy the axioms of topology:

ϕ_1 : If null edge set and the complete edge set E must belong to the class D .



As $\phi, E \in D$, thus first axiom of topology is satisfied.

ϕ_2 : To satisfy this axiom of topology, we have to satisfy that the union of all the subsets of the class D lies in D itself.

As it is shown earlier that union of a null edge set to another edge set is the edge set itself.

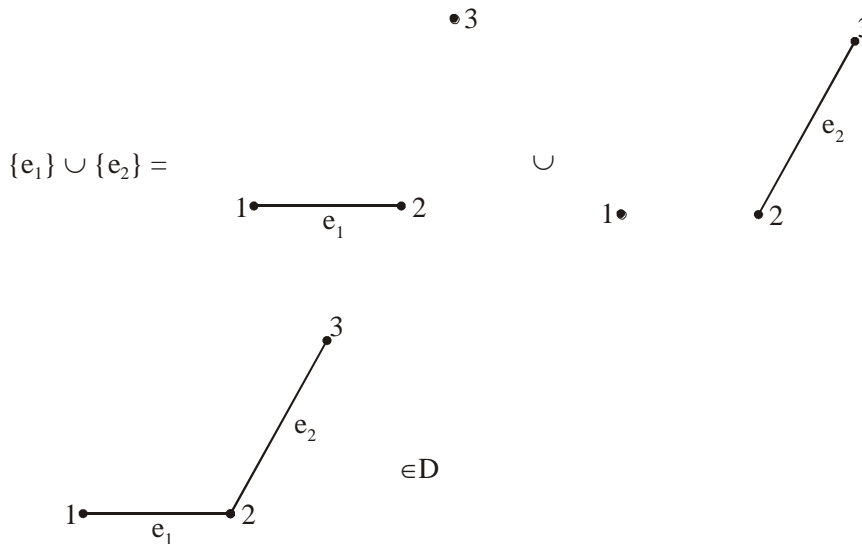
So it is quite obvious that

$\phi \cup \{e_1\} \in D, \phi \cup \{e_2\} \in D, \{e_2\} \cup E \in D, \{e_1\} \cup \{e_2\} \cup E \in D,$

Similarly the union of the edge set E with any set will be the set E itself. Hence union of E belongs to D .

Thus $\{e_1\} \in D, \{e_2\} \in D, E \cup \{e_1\} \in D, E \cup \{e_2\} \in D,$

Only to check $\{e_2\} \cup \{e_1\}$ belongs to D or not !



hence union of all the subset lies in D itself.

\square_3 : This is the last axiom, here we have to show that the intersection of all the subsets must lie in itself D. First the intersection of null edge set to any other edge subset is null set itself, hence it must lie in the class D itself.

$$D \text{ and } \in \{e_2\} \cap e_1 \} \{ \cap \phi D, \in \{e_2\} \cap \phi D, \in \{e_1\} \cap \phi \in E \cap e_2 \} \{ \cap e_1 \} \{ \cap \phi$$

CONCLUSION

We wish to examine the concepts of algebra and analysis of the set of graphs. In our work we find it very easily. All the conditions of algebra and analysis are satisfied on the set of graphs. By the meaning set of graphs we meant only the edge set of a simple graph. We have developed the new approach of algebra and analysis of the set of graphs in a very simple and interesting manner.

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